Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER					

2004404347

FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **May/June 2023**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (LK) 327441
© UCLES 2023

BLANK PAGE

1 **(a)** Show that the system of equations

$$\begin{aligned} x+2y+3z &= 1, \\ 4x+5y+6z &= 1, \\ 7x+8y+9z &= 1, \end{aligned}$$

does not have a unique solution. [2]

.....

.....

.....

.....

.....

.....

.....

.....

(b) Show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

2 Use the substitution $z = x + y$ to find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1 + 3x + 3y}{3x + 3y - 1}$$

for which $y = 0$ when $x = 1$. Give your answer in the form $a\ln(x+y) + b(x-y) + c = 0$, where a, b and c are constants to be determined. [7]

3 **(a)** By considering the binomial expansion of $(z+z^{-1})^4$, where $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that $\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Use the substitution $x = \sin\theta$ to find the exact value of $\int_0^{\frac{1}{2}} (1-x^2)^{\frac{3}{2}}\,\mathrm{d}x$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

4 The integral I_n is defined by $I_n = \int_0^1 (1+x^5)^n \, \mathrm{d}x$.

(a) By considering $\frac{\mathrm{d}}{\mathrm{d}x}\left(x(1+x^5)^n\right)$, or otherwise, show that

$$(5n+1)I_n = 2^n + 5nI_{n-1}.$$ [5]

(b) Find the exact value of I_3. [4]

5 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 18 & 5 & -11 \\ 8 & 6 & -4 \\ 32 & 10 & -20 \end{pmatrix}.$$

(a) Show that the characteristic equation of **A** is $\lambda^3 - 4\lambda^2 - 20\lambda + 48 = 0$ and hence find the eigenvalues of **A**. [4]

(b) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{PDP}^{-1}$. [6]

6 Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 36x = 37\sin t,$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$. [11]

7 **(a)** Use the substitution $u = x^2 - 1$ to find $\int \frac{x}{\sqrt{x^2-1}}\,dx$. [3]

.....

.....

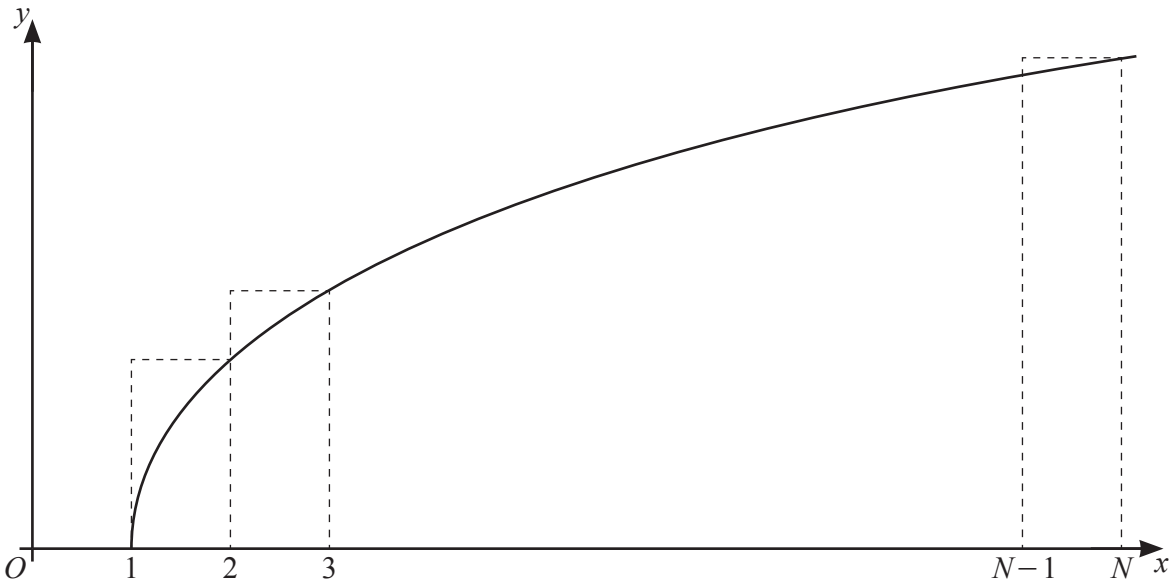
.....

.....

.....

.....

.....



The diagram shows the curve with equation $y = \cosh^{-1}x$ together with a set of $(N-1)$ rectangles of unit width.

(b) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=2}^{N} \ln\left(r + \sqrt{r^2-1}\right) > N\ln\left(N + \sqrt{N^2-1}\right) - \sqrt{N^2-1}.$$ [5]

.....

.....

.....

.....

.....

.....

(c) Use a similar method to find, in terms of N, an upper bound for $\sum_{r=2}^{N} \ln\left(r+\sqrt{r^2-1}\right)$. [3]

8 **(a)** Starting from the definitions of sech and tanh in terms of exponentials, prove that

$$1 - \operatorname{sech}^2 t = \tanh^2 t.$$ [3]

.....

.....

.....

.....

.....

.....

.....

.....

The curve C has parametric equations

$$x = \tfrac{1}{2}\tanh^2 t + \ln \operatorname{sech} t, \qquad y = 1 + \tanh^4 t, \qquad \text{for } t > 0.$$

(b) Show that $\dfrac{dy}{dx} = -4\operatorname{sech}^2 t$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the coordinates of the point on C with $\frac{d^2y}{dx^2} = -\frac{9}{2}$, giving your answer in the form $(a + \ln b, c)$ where a, b and c are rational numbers. [6]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

© UCLES 2023